Thornton & Marion 5th Edition Problem 3 – 11

Reproduce Figure 3-8 in the text for the energy and rate of energy loss of the lightly damped harmonic oscillator.

The position and velocity of the oscillator are given by

In[1]:= **x[t_] = A*Exp[−β*t] Cos[ω$_S$*t − δ]**

Out[1]= $A\, e^{-t\beta} \text{Cos}[\delta - t\, \omega_S]$

In[2]:= **s = x[t]**

Out[2]= $A\, e^{-t\beta} \text{Cos}[\delta - t\, \omega_S]$

To simplify, assume that *x(t =0) =A*, so that δ = 0. Thus

In[3]:= **v = x'[t]**

Out[3]= $-A\, e^{-t\beta}\, \beta\, \text{Cos}[\delta - t\, \omega_S] + A\, e^{-t\beta}\, \text{Sin}[\delta - t\, \omega_S]\, \omega_S$

To simplify, assume that *x(t =0) =A*, so that δ = 0. Then the Energy is the sum of

In[4]:= **δ = 0;**

$$U = \left(\frac{k}{2}\right) s^2$$

$$T = \left(\frac{m}{2}\right) v^2$$

Out[5]= $\frac{1}{2} A^2\, e^{-2\,t\,\beta}\, k\, \text{Cos}[t\, \omega_S]^2$

Out[6]= $\frac{1}{2} m \left(-A\, e^{-t\beta}\, \beta\, \text{Cos}[t\, \omega_S] - A\, e^{-t\beta}\, \text{Sin}[t\, \omega_S]\, \omega_S\right)^2$

In[7]:= **Etot = U + T**

Out[7]= $\frac{1}{2} A^2\, e^{-2\,t\,\beta}\, k\, \text{Cos}[t\, \omega_S]^2 + \frac{1}{2} m \left(-A\, e^{-t\beta}\, \beta\, \text{Cos}[t\, \omega_S] - A\, e^{-t\beta}\, \text{Sin}[t\, \omega_S]\, \omega_S\right)^2$

In[9]:= **Expand[Etot]**

Out[9]= $\frac{1}{2} A^2\, e^{-2\,t\,\beta}\, k\, \text{Cos}[t\, \omega_S]^2 + \frac{1}{2} A^2\, e^{-2\,t\,\beta}\, m\, \beta^2\, \text{Cos}[t\, \omega_S]^2 +$

$A^2\, e^{-2\,t\,\beta}\, m\, \beta\, \text{Cos}[t\, \omega_S]\, \text{Sin}[t\, \omega_S]\, \omega_S + \frac{1}{2} A^2\, e^{-2\,t\,\beta}\, m\, \text{Sin}[t\, \omega_S]^2\, \omega_S^2$

In[10]:= **Simplify[Etot]**

Out[10]= $\frac{1}{2} A^2\, e^{-2\,t\,\beta} \left(\left(k + m\, \beta^2\right) \text{Cos}[t\, \omega_S]^2 + m\, \beta\, \text{Sin}[2\, t\, \omega_S]\, \omega_S + m\, \text{Sin}[t\, \omega_S]^2\, \omega_S^2\right)$

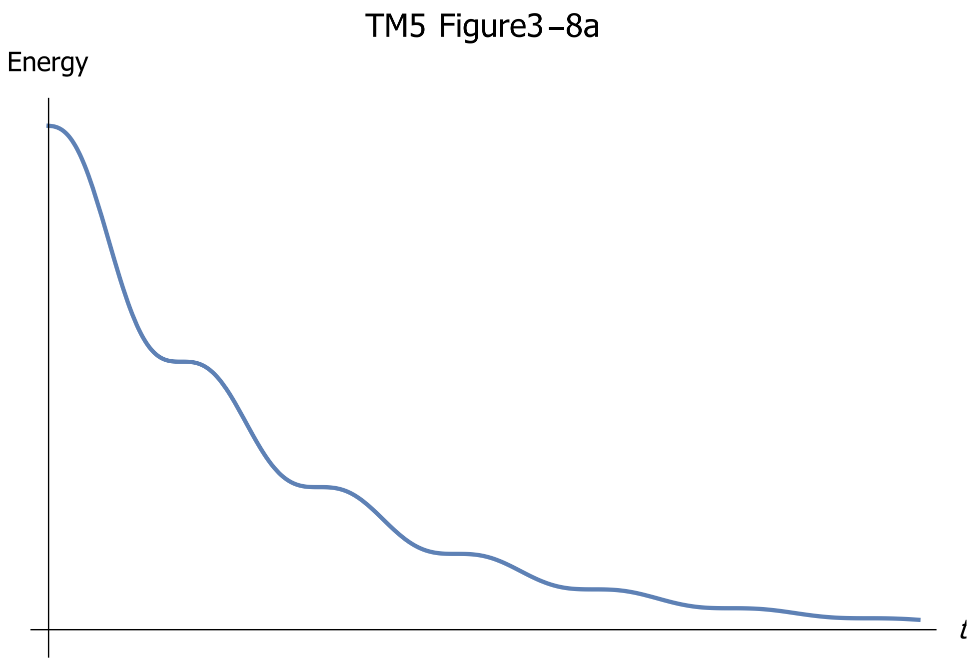
To plot this, set the constants for the case of light damping, $\omega_0^2 > \beta^2$

In[11]:=
```
ωN = 1;
β = 0.1;
ωS = Sqrt[ωN^2 - β^2];
m = 2;
k = 2;
A = 1;
Etot
```

Out[17]= $e^{-0.2\,t}\,\mathrm{Cos}[0.994987\,t]^2 + \left(-0.1\,e^{-0.1\,t}\,\mathrm{Cos}[0.994987\,t] - 0.994987\,e^{-0.1\,t}\,\mathrm{Sin}[0.994987\,t]\right)^2$

In[18]:=
```
Plot[Etot, {t, 0, 20},
  AxesLabel → {t, Energy},
  BaseStyle → {FontFamily → Arial, 10, FontColor → RGBColor[0, 0.5, 0]},
  Ticks → None,
  PlotLabel → "TM5 Figure3−8a"]
```

Out[18]=



Take the derivative to find the rate of energy loss and plot it.

In[19]:=
```
rate = D[Etot, t]
```

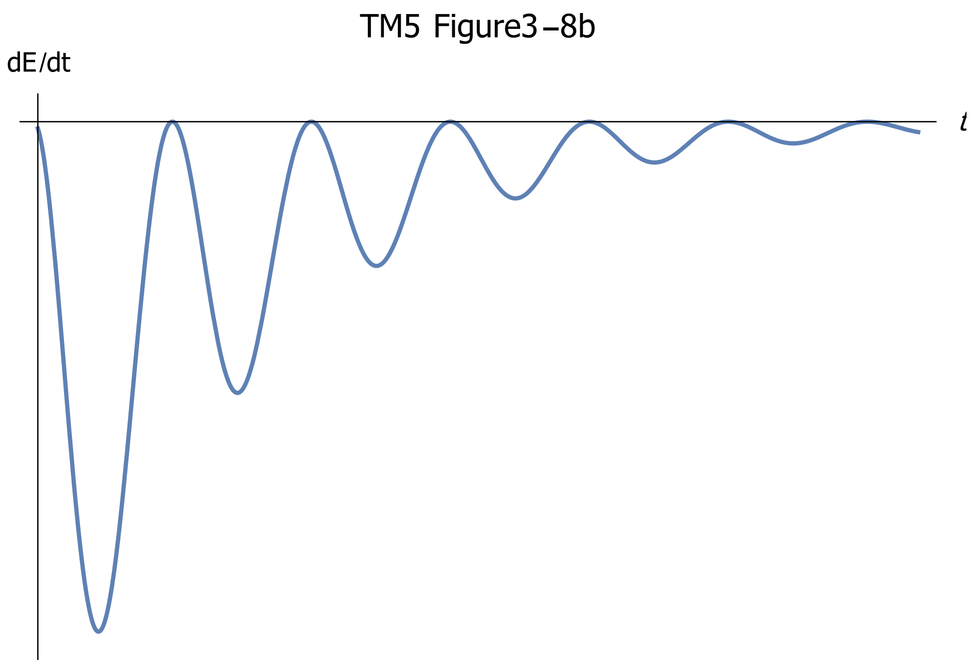
Out[19]= $-0.2\,e^{-0.2\,t}\,\mathrm{Cos}[0.994987\,t]^2 - 1.98997\,e^{-0.2\,t}\,\mathrm{Cos}[0.994987\,t]\,\mathrm{Sin}[0.994987\,t] + 2\left(-0.1\,e^{-0.1\,t}\,\mathrm{Cos}[0.994987\,t] - 0.994987\,e^{-0.1\,t}\,\mathrm{Sin}[0.994987\,t]\right)\left(-0.98\,e^{-0.1\,t}\,\mathrm{Cos}[0.994987\,t] + 0.198997\,e^{-0.1\,t}\,\mathrm{Sin}[0.994987\,t]\right)$

Assume the same values of the constants to plot it

In[20]:=
$\omega_N = 1;$
$\beta = 0.1;$
$\omega_S = \sqrt{\omega_N^2 - \beta^2};$
$m = 2;$
$k = 2;$
$A = 1;$
rate

Out[26]= $-0.2\, e^{-0.2\,t} \text{Cos}[0.994987\, t]^2 - 1.98997\, e^{-0.2\,t} \text{Cos}[0.994987\, t]\, \text{Sin}[0.994987\, t] + 2\left(-0.1\, e^{-0.1\,t} \text{Cos}[0.994987\, t] - 0.994987\, e^{-0.1\,t} \text{Sin}[0.994987\, t]\right) \left(-0.98\, e^{-0.1\,t} \text{Cos}[0.994987\, t] + 0.198997\, e^{-0.1\,t} \text{Sin}[0.994987\, t]\right)$

In[27]:=
```
Plot[rate, {t, 0, 20},
  PlotRange → All,
  BaseStyle → {FontFamily → Arial, FontSize → 10, FontColor → RGBColor[0, 0.5, 0]},
  AxesLabel → {t, "dE/dt"},
  Ticks → None,
  PlotLabel → "TM5 Figure3-8b"]
```

Out[27]=


In[28]:=
```
Export["TM5Pr03_11Mathematica.pdf", SelectedNotebook[]]
```